

带非线性扰动多时滞系统的鲁棒稳定

罗 亮,金朝永,陈德银,胡南辉

(广东工业大学 应用数学学院,广东 广州 510006)

摘要: 研究一类带有非线性扰动的不确定多时滞系统的鲁棒稳定问题. 基于李雅普诺夫定理,给出了系统多时滞相关的稳定准则,再通过对闭环系统进行稳定分析,推导出状态反馈控制的充分条件,该条件进一步等价地转化为一组线性矩阵不等式的可解性问题. 通过两个例子说明文中所提出的方法可减少结果的保守性,证明文中鲁棒控制方法的有效性.

关键词: 非线性扰动;状态反馈;时滞依赖;鲁棒稳定

中图分类号: TP271

文献标识码: A

文章编号: 1007-7162(2008)03-0047-04

时滞现象普遍存在于实际应用的控制系统中,并且常常是导致系统不稳定的重要因素. 此问题已受到国内外学者的广泛关注,并取得了许多研究成果^[1-5]. 时滞系统控制理论一般分为两类:一类是时滞独立稳定^[6-7];另一类是时滞相关稳定控制^[8-9]. 但时滞相关稳定控制处于初级阶段,且时滞相关控制的结论比时滞独立控制具有较小的保守性. 因此研究时滞相关的稳定性和鲁棒稳定判据显得非常重要. 然而,一些文献没有考虑耦合的非线性扰动和控制对时滞的控制能力^[10-11].

本文研究了非线性扰动时滞系统的稳定性和稳定控制问题. 通过建立系统时滞相关鲁棒稳定性判据,指出系统时滞相关鲁棒稳定时存在时滞上界,并给出了时滞相关的鲁棒稳定控制器存在判据和控制器设计方法.

1 系统的描述和准备工作

考虑下述带有非线性摄动的多时滞系统:

$$\begin{aligned} \dot{x}(t) = & Ax + \sum_{i=1}^j Ax(t-i) + Bu(t) + \\ & f(t, x(t), x(t-i_1), \dots, x(t-i_j)), \end{aligned} \quad (1)$$

其中 $x(t) \in R^n$ 是系统的状态, $u(t) \in R^{n_u}$ 是系统的控制输入, A, A_i, B 分别为适当维数的已知常矩阵; $\in R^+$ 为未知有界常时滞,满足 $0 \leq i \leq M$, M 是时滞的已知上界 ($i=1, 2, \dots, j$). 非线性摄动 $f(t, x(t), x(t-i_1), \dots, x(t-i_j))$ 为时变的含有状态和时滞状态的耦合函数,其结构为

$$\begin{aligned} f^T(t, x(t), x(t-i_1), \dots, x(t-i_j)) f(t, x(t), \\ x(t-i_1), \dots, x(t-i_j)) \leq & \alpha_0^2 x^T(t) G_0^T G_0 x(t) + \\ & \alpha_1^2 x^T(t-i_1) G_1^T G_1 x(t-i_1) + \dots + \\ & \alpha_j^2 x^T(t-i_j) G_j^T G_j x(t-i_j), \end{aligned} \quad (2)$$

其中 α_i 是已知正的常数, G_i 是已知定常的结构矩阵 ($i=1, 2, \dots, j$).

引理 1 设 $a(\cdot) \in R^{n_a}$, $b(\cdot) \in R^{n_b}$ 和 $N(\cdot) \in R^{n_a \times n_b}$ 是区间 Φ 上的可积函数. 则对任意满足 $M(X, Y, Z) \equiv \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$ 的矩阵 $X \in R^{n_a \times n_a}$, $Y \in R^{n_a \times n_b}$ 和 $Z \in R^{n_b \times n_b}$, 有

$$\begin{aligned} -2 \int_{\Phi} a^T(s) N(s) b(s) ds \leq \\ \int_{\Phi} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix} ds \end{aligned} \quad (3)$$

引理 2^[12] 设 $\Phi_0(x)$ 和 $\Phi_1(x)$ 是 R^n 上的两个任意的二次型函数. 若存在 $\rho > 0$ 使得不等式 $\Phi_0(x) - \rho \Phi_1(x) < 0$ 对任意非零向量 $x \in R^n$ 都成立, 则不等式 $\Phi_0(x) < 0$ 对所有满足 $\Phi_1(x) \leq 0$ 的任意 $x \in R^n - \{0\}$ 都成立.

2 主要结论

2.1 考虑系统 (1) 的自治系统 (即 $u(t) \equiv 0$) 有以下结果

定理 1 对于正数 $M > 0$, 如果存在对称正定矩阵 $\tilde{P}, \tilde{X}, \tilde{Y}, \tilde{Z}$, $\rho > 0$ 使得式 (4)、(5) 成立.

收稿日期: 2008-02-27

作者简介: 罗 亮 (1981-), 女, 硕士研究生, 主要研究方向为非线性系统的鲁棒控制.

$$\begin{bmatrix} \tilde{\Psi} + \alpha_0^2 G_0^T G_0 & L & \tilde{P} + j_M A^T \tilde{Z} & j_M A^T \tilde{Z} \\ L^T & \Gamma & \Theta^T & \Theta^T \\ \tilde{P} + j_M \tilde{Z} A & \Theta & j_M \tilde{Z} & 0 \\ j_M \tilde{Z} A & \Theta & 0 & -j_M \tilde{Z} \end{bmatrix} < 0, \quad (4)$$

$$M(\tilde{X}, \tilde{Y}, \tilde{Z}) \geq 0 \quad (5)$$

其中

$$\begin{aligned} \tilde{P} &= P/\rho, \tilde{X} = X/\rho, \tilde{Y} = Y/\rho, \tilde{Z} = Z/\rho, \\ \tilde{\Psi} &= A^T \tilde{P} + \tilde{P} A + j_M \tilde{X} + \tilde{Y} + \tilde{Y}^T + \tilde{Q}, \\ L &= [\tilde{P} A_1 - \tilde{Y}, \tilde{P} A_2 - \tilde{Y}, \dots, \tilde{P} A_j - \tilde{Y}], \\ \Gamma &= \text{diag}(-\tilde{Q} + \alpha_1^2 G_1^T G_1, -\tilde{Q} + \alpha_2^2 G_2^T G_2, \dots, -\tilde{Q} + \alpha_j^2 G_j^T G_j), \\ \Theta &= [j_M \tilde{Z} A_1, j_M \tilde{Z} A_2, \dots, j_M \tilde{Z} A_j], \end{aligned}$$

则对于任意 $i \in [0, j_M]$, $i = 1, 2, \dots, j$ 系统 (1) ($u(t) \equiv 0$) 是鲁棒稳定的.

证明:

记 $f(t) = f(t, x(t), x(t-1), \dots, x(t-i))$, $i = 1, 2, \dots, j$ 将系统 (1) ($u(t) \equiv 0$) 写成式 (6) 的形式:

$$\dot{x}(t) = \left(A + \sum_{i=1}^j A_i \right) x(t) - \sum_{i=1}^j A_i \int_{t-i}^t \dot{x}(s) ds + f(t). \quad (6)$$

取 Lyapunov 泛函

$$V(t, x(t)) = x^T(t) P x(t) + \sum_{i=1}^j \int_{t-i}^t \int_{t-s}^t (\mu) Z x(\mu) d\mu ds + \sum_{i=1}^j \int_{t-i}^t x^T(s) Q x(s) ds, \quad (7)$$

则沿系统的解:

$$\begin{aligned} \dot{V}(t, x(t)) &\leq x^T(t) \left(A + \sum_{i=1}^j A_i \right)^T P x(t) + \\ &x^T(t) P \left(A + \sum_{i=1}^j A_i \right) x(t) + 2x^T(t) P f(t) - \\ &2x^T(t) P \sum_{i=1}^j A_i \int_{t-i}^t \dot{x}(s) ds + j_M \dot{x}(t) Z x(t) - \\ &\sum_{i=1}^j \int_{t-i}^t \dot{x}(s) ds + j_M x^T(t) Q x(t) - \\ &\sum_{i=1}^j x^T(t-i) Q x(t-i). \end{aligned} \quad (8)$$

由引理 1 得

$$-2x^T(t) P \sum_{i=1}^j A_i \int_{t-i}^t \dot{x}(s) ds \leq$$

$$\begin{aligned} &\sum_{i=1}^j \int_{t-i}^t \int_{t-s}^t x^T(s) \begin{bmatrix} X & Y - PA_i \\ Y^T - A_i^T P & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(s) \end{bmatrix} ds \leq \\ &j_M x^T(t) X x(t) + 2 \sum_{i=1}^j x^T(t) (Y - PA_i) \int_{t-i}^t \dot{x}(s) ds + \\ &\sum_{i=1}^j \int_{t-i}^t x^T(s) Z x(s) ds, \end{aligned}$$

记向量 $z = [x^T(t), x^T(t-1), \dots, x^T(t-j)]$, $f^{T(0)T}$ 综合以上结论, 有

$$V \leq z^T \begin{bmatrix} \Psi_0 & L_0 & j_M A^T Z + \tilde{P} \\ L_0^T & \Gamma_0 & \Theta_0^T \\ P + j_M Z A & \Theta_0 & j_M Z \end{bmatrix} z, \quad (9)$$

其中,

$$\begin{aligned} \Psi_0 &= A^T P + P A + j_M X + \tilde{Y} + \tilde{Y}^T + \tilde{Q} + j_M A^T Z A, \\ \Theta_0 &= [j_M Z A_1, j_M Z A_2, \dots, j_M Z A_j], \\ L_0 &= [P A_1 - Y + j_M A^T Z A_1, P A_2 - Y + \\ &j_M A^T Z A_2, \dots, P A_j - Y + j_M A^T Z A_j], \end{aligned}$$

$$\Gamma_0 = \begin{bmatrix} -Q + j_M A_1^T Z A_1 & j_M A_1^T Z A_2 & \dots & j_M A_1^T Z A_j \\ j_M A_2^T Z A_1 & -Q + j_M A_2^T Z A_2 & \dots & j_M A_2^T Z A_j \\ \dots & \dots & \ddots & \dots \\ j_M A_j^T Z A_1 & j_M A_j^T Z A_2 & \dots & -Q + j_M A_j^T Z A_j \end{bmatrix}.$$

将不等式 (9) 的右边函数记为 $\Phi_0(z)$, 而对于约束不等式 (2), 则可以改写为下面二次型函数的不等式

$$\Phi_1(z) = z^T \cdot \text{diag}\{-\alpha_0^2 G_0^T G_0 - \alpha_1^2 G_1^T, \dots, -\alpha_j^2 G_j, I\} \cdot z \leq 0 \quad (10)$$

由引理 2 知道, 如果存在 $\rho > 0$, 使得下面矩阵不等式组 (11) - (12) 成立, 则对满足非线性摄动范围 (10) 的任意 $x \in R^n - \{0\}$, 有 $V \leq \Phi_0(z) < 0$

$$\begin{aligned} \Phi_0(z) - \rho \Phi_1(z) &= \\ \begin{bmatrix} \Psi_0 + \alpha_0^2 \rho G_0^T G_0 & L_0 & j_M A^T Z + \tilde{P} \\ L_0^T & \Gamma_0 & \Theta_0^T \\ P + j_M Z A & \Theta_0 & j_M Z \end{bmatrix} &< 0, \end{aligned} \quad (11)$$

$$M(x, y, Z) \geq 0, \quad (12)$$

其中 Ψ_0, L_0, Θ_0 同上,

而 $\tilde{\Gamma}_0 = \Gamma_0 + \text{diag}\{\alpha_1^2 \rho G_1^T G_1, \alpha_2^2 \rho G_2^T G_2, \dots, \alpha_j^2 \rho G_j^T G_j\}$. 应用 Schur 补引理, 矩阵不等式组 (11)、(12) 等价变形为不等式组 (4)、(5), 故 $V < 0$ 证毕.

2.2 时滞状态反馈控制情形

下面考虑非线性时滞系统 (1) 经时滞状态反馈的时滞相关鲁棒稳定控制问题.

$$\dot{x}(t) = (A + Bk)x(t) + \sum_{i=1}^j (A_i + Bk_i)x(t-i) +$$

$$f(t, x(t), x(t-1), \dots, x(t-j)). \quad (13)$$

定理 2 对于非线性时滞系统 (1)和给定的非线性摄动范围 (2),如果矩阵不等式 (14)有可行解:

$$\begin{bmatrix} \Delta & \Xi & V + j_M (PA^T + N^T B^T) & j_M (RA^T + N^T B^T) & \alpha_0 R G_0^T & 0 \\ \Xi^T & -\Lambda & M^T & M^T & 0 & H^T \\ V + j_M (AR + BN) & M & j_M V - V^2 & 0 & \dots & 0 \\ j_M (AR + BN) & M & 0 & -j_M V & 0 & 0 \\ \alpha_0 G_0 R & 0 & \dots & 0 & I & 0 \\ 0 & H & 0 & 0 & 0 & I \end{bmatrix} < 0, \quad (14)$$

$$u(t) = kx(t) + \sum_{i=1}^j k_i x(t-i) \equiv NR^{-1}x(t) + \sum_{i=1}^j N_i R^{-1}x(t-i). \quad (15)$$

其中,

$$\begin{aligned} \Delta &= RA^T + AR + j_M K + JS + N^T B + BN, \\ \Xi &= A_1 R + BN_1, A_2 R + BN_2, \dots, A_j R + BN_j, \\ \Lambda &= \text{diag}\{S, S, \dots, S\}, \\ M &= j_M (A_1 R + BN_1), j_M (A_2 R + BN_2), \dots, \\ & j_M (A_j R + BN_j), \\ H &= \text{diag}\{\alpha_1 G_1 R, \alpha_2 G_2 R, \dots, \alpha_j G_j R\}, I = \text{diag}\{I, I, \dots, I\}, \end{aligned}$$

则对任意满足 $0 \leq i \leq m$ 的时滞 i , 时滞状态反馈式 (15)是系统 (1)鲁棒稳定控制器.

证明:在式 (5) 中,选取 $\tilde{X} > 0$, $\tilde{Y} \equiv 0$ 和 $\tilde{Z} > 0$

在式 (4) 中分别用 $(A + Bk)$ 代替 A , $\sum_{i=1}^j (A_i + Bk_i)$

代替 $\sum_{i=1}^j A_i x(t-i)$, 整理之后再对式 (11) 的两边同

时乘以 $\text{diag}\{\tilde{P}^{-1}, \tilde{P}^{-1}, \dots, \tilde{P}^{-1}, \tilde{Z}^{-1}, \tilde{Z}^{-1}\}$.

令 $R = \tilde{P}^{-1}$, $V = \tilde{Z}^{-1}$, $S = \tilde{P}^{-1} Q \tilde{P}^{-1} - 1$, $K = \tilde{P}^{-1} X \tilde{P}^{-1}$ 和 $N = k \tilde{P}^{-1}$, $N_i = k_i \tilde{P}^{-1}$ ($i = 1, 2, \dots, j$), 利用 Schur 补引理即可证毕.

另外,引理 1 在正定矩阵意义下和其他不变的情况下, Z 越小, 右式作为左式的上界则保守性越小. 针对不等式 (14) 中的非线性项 $j_M V - V^2$, 可以取 $V - \delta I$, $\delta > 0$ 只要 δ 足够大 ($\delta > m$), 就可以保证 $j_M V - V^2 < 0$, 且使不等式 (14) 变成 LMI

3 数值例子

例 1

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} x(t-1) + f(t, x(t), x(t-1)), \quad (16)$$

正定矩阵 R, S, V, K , 矩阵 N, N_i ($i = 1, 2, \dots, j$) 和正数 $m > 0$,

$$\text{当 } B = 0, G_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0.8 \end{bmatrix}, G_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \alpha_0 =$$

$\alpha_1 = 0.3$ 时, 由定理 1, 得系统 (16) ($u(t) \equiv 0$) 的时滞相关鲁棒稳定时滞上界: $m = 1.4981$.

而文献 [13 - 15] 得到的分别为: < 0.4283 , < 0.2454 , < 0.1614 , 可见本文的结果保守性更少.

例 2

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 0 \\ 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix} x(t-1) + \\ & \begin{bmatrix} 1 & -1 \\ -0.4 & 2 \end{bmatrix} x(t-2) + \begin{bmatrix} -1.1 \\ 1 \end{bmatrix} u(t) + f(t, x(t), \\ & x(t-1), x(t-2)), \end{aligned} \quad (17)$$

$$\text{其中 } G_0 = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}, G_1 = \begin{bmatrix} 1 & 0 \\ 0.3 & 1 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}.$$

$\alpha_0 = \alpha_1 = \alpha_2 = 0.8$ 当 $V = \text{diag}\{10, 10\}$, 由定理 2 得状态反馈: $u(t) = [31 \ 126 \ -1 \ 734] x(t) + [0.7076 \ 0.1088] x(t-1) + [-0.0643 \ -1.3614] x(t-2)$, 对任意时滞: $0 \leq i \leq m = 1.852$ ($i = 1, 2$) 系统 (17) 都是鲁棒镇定.

4 结语

主要研究了一类带有非线性扰动的时滞系统多时滞依赖型鲁棒控制器的设计问题, 通过构造状态反馈控制器得到多时滞依赖型 LMI, 进而给出时滞系统的多时滞依赖型鲁棒稳定的判断方法. 从而减弱了控制器设计的保守性, 使得该方法具有更广泛的使用范围.

参考文献:

- [1] Xi L, De S, Carlos E. Criteria for robust stability and stabilization of uncertain linear systems with state delay[J]. Automatica, 1997, 33(9): 1657-1662
- [2] Lee C H, Su T H, Kung F C. On the robustness of stability

- for uncertain time-delay systems[J]. *Int J Systems Sci* 1995, 26(2): 457-465.
- [3] Xu B. Delay-dependent stability for linear uncertain time-delay systems[J]. *Control Theory and Applications*, 1996, 13(4): 426-431.
- [4] Xu B. Stability robustness bounds for linear systems with multiple time varying delayed perturbations[J]. *Int J Systems Sci*, 1997, 28(12): 1311-1317.
- [5] Su T J, Huang C G. Robust stability of delay dependence for linear uncertain systems[J]. *IEEE Trans A C*, 1992, 237(10): 1656-1659.
- [6] Su J H. Further results on the robust stability of linear systems with a single time delay[J]. *Systems and Control Letters*, 1994, 23(6): 375.
- [7] Cao Y Y, Jam E S L. Computation of robust stability bounds for time-delay systems with nonlinear time-varying perturbations[J]. *International Journal of Systems Science*, 2000, 31(3): 359.
- [8] Park P. A delay-dependent stability criterion for systems with uncertain time-invariant delay[J]. *IEEE Transactions on Automatic Control*, 1999, 44(4): 876.
- [9] Moon Y S. Delay-dependent robust stabilization of uncertain state-delayed systems[J]. *International of Control*, 2001, 74(14): 1447.
- [10] Niculescu S. Robust stability and stabilization of uncertain linear systems with state delay: Single delay case[C] // *IFAC Symposium on Robust Control Design*, Brazil: Rio de Janeiro, 1994.
- [11] Li X, De Souza C E. Criteria and stabilization of uncertain linear systems with state delays[J]. *Automatic*, 1997, 33(9): 1657.
- [12] Yakubovich V A. *Procedure in non-linear control theory* [J]. *Vestnik Leningrad Univ*, 1971, 1: 62.
- [13] Ma Xin-jun, Xu B. Delay-dependent robust stability of uncertain systems with multiple Time-delay and nonlinear perturbation[J]. *Journal of South China University of Technology: Natural Science Edition*, 2004, 32(10): 36-40.
- [14] 陈尔彦,徐世杰. 时滞不确定系统的时滞相关稳定性[J]. *哈尔滨工业大学学报*, 2001, 33(6): 8192-821.
- [15] Su T J, Huang C G. Robust stability of delay dependence for linear uncertain systems[J]. *IEEE Trans Auto Contr*, 1992, AC237(10): 1656-1659.

Robust Stability of Multiple Time-delay Systems with Nonlinear Perturbations

Luo Liang, Jin Chao-yong, Chen De-yin, Hu Nan-hui

(Faculty of Applied Mathematics, Guangdong University of Technology, Guangzhou 510006, China)

Abstract: The problem with robust stabilization of uncertain multiple time-delay systems with nonlinear perturbation is proposed. Firstly, based on Lyapunov stability theory, the delay dependent stabilization criteria for the system are set. Secondly, robust controller is given by analyzing the stability of class-loop system. The sufficient condition for the existence of the state feedback controller is derived in terms of linear matrix inequalities. It is finally illustrated that the method presented can reduce the conservativeness of the results, which indicates the validity of the proposed method.

Key words: nonlinear perturbation; state feedback; delay dependent; robust stability